Overview. I am an abstract homotopy theorist/∞-category theorist with a particular interest in functor calculi, both in their own right and their applications to representation stability theory.

I have two main ongoing and interrelated research projects. In the first of these, "corepresentation functor calculus," generalize techniques from my dissertation to develop an axiomatic framework which encompasses the stable case of a number of flavors of functor calculus considered in the literature and furnishes new and interesting functor calculi for, to the best of my knowledge, all 1-categories considered in the representation stability literature. A major goal is to establish that representation stability can be understood as an emanation of functor calculus and to supply new tools and perspectives from functor calculus for the study of representation stability.

In my second main research project, I develop an abstract theory of functor calculi in an arbitrary ∞-equipment using the theory developed by Ruit in [Rui25; Rui24]. This level of generality accommodates the significant role played by enrichment in corepresentation calculus and representation stability; is able to encompass recently introduced flavors of functor calculi for G-∞-categories pertinent to equivariant homotopy theory such as Dotto's equivariant Goodwillie calculus, developed in [DM16; Dot16], and Bhattacharya and Hu's equivariant Weiss calculus, developed in [BH24]; and facilitates the future development of functor calculi for enriched ∞-categories, internal ∞-categories, etc.

FI-calculus. In my dissertation [Arr23], I introduce a new flavor of functor calculus, "FI-calculus", for functors, which I call FI-objects, from FI, the category of finite sets and injections, to an arbitrary stable presentable ∞-category. This FI-calculus enjoys an array of interesting qualities, some of which it shares with its relative, Weiss' orthogonal calculus, and some of which are, among functor calculi appearing in the literature, distinctive to it.

Among these distinctive features is the property that the Taylor coefficients classifying the homogeneous layers of a Taylor tower, when considered in aggregate, themselves carry, in a natural way, the structure of an FI-object. I show that specifying an FI-object (or formal Taylor tower) with a given FI-object of Taylor coefficients depends on solving a family of lifting problems along norm maps obtained from the coefficients. It follows that in an ∞-category in which all Tate constructions vanish, the aggregate Taylor coefficient construction determines an equivalence between formal Taylor towers and FI-objects which restricts to an equivalence between analytic FI-objects and torsion FI-objects.

Perhaps the most compelling facet of FI-calculus is that it ∞-categorifies representation stability for FI-modules. More precisely stated, polynomial FI-objects valued in the ∞-category of chain complexes of R-modules for R a Noetherian ring have representation stable homology, and conversely, representation stable FI-modules understood as FI-chain complexes concentrated in degree 0 agree with a unique polynomial FI-chain complex when restricted to a full subcategory of FI spanned by all sufficiently large sets.

This relationship has several implications for the study of representation stability of FI-modules. First, it shows that the study of representation stability may reasonably be understood to encompass not just representation stable FI-modules but also analytic FI-chain complexes, which enjoy many of the same desirable algebraic and categorical properties as representation stable FI-modules. Second, via the higher order homology operations on analytic FI-chain complexes, FI-calculus provides a notion of (one type of) higher-order representation stability in the otherwise "pre-stable" range of an FI-chain complex. Third, the category theoretic approach of FI-calculus offers useful perspectives on representation stability in (∞-)categories other than those which are $\mathbb{Q}$-linear – e.g. by establishing an equivalence between analytic and torsion FI-objects valued in $K(n)$-local spectra.

In pursuit of the second of these applications, I have completely calculated the homology (and higher order homology operations) of the indecomposable homogeneous rational FI-chain complexes. This new result will appear in an updated version of [Arr23] and facilitates the calculation of higher-order representation stability phenomena from the Taylor coefficients of a rational FI-chain complex. In the course of these calculations, I have also identified torsion in the integral homology of homogeneous FI-objects associated to Specht modules. This torsion is a tantalizing glimpse of modular version of representation stability, where the usual rule relating Young diagrams is replaced by a more mysterious and complex pattern.

Computational topology. In order to begin to investigate the homology groups described above, I wished to examine some examples, but soon found almost all of the relevant simplicial complexes to be intractably large and complicated. Existing software for speedily computing the homology groups of simplicial complexes relies largely on the discrete Morse theory of Forman (see [For02] for an overview). However, these algorithms generally require that all simplices (rather than just the maximal simplices) of the simplicial complex be calculated and stored in memory at the beginning of the computation. For some of the large complexes of interest to me, this would have required terabytes of memory.

To circumvent the problem, I have written an efficient program in Rust which only stores the maximal simplices and edges of a simplicial complex in memory and uses the link condition described in [ALS11] to efficiently contract edges while preserving the homotopy type of the complex. In my testing, the speed of my algorithm is comparable to that of Vidit Nanda's Perseus, [Nan], a discrete Morse theory algorithm written in C++, while requiring orders of magnitude less memory.

Corepresentation calculus. In my current project, I axiomatize the properties of FI which give rise to the existence of a functor calculus for FI, which I call a corepresentation calculus on an ∞-category $\mathcal{C}$. Given a small ∞-category $\mathcal{C}$, one identifies a family Π of full subcategories $\mathcal{C}_\alpha \subseteq \mathcal{C}$ closed under intersection such that for each $\mathcal{C}_\alpha$ in Π and each $a \in \mathcal{C}$, the comma ∞-category $\mathcal{C}_\alpha \downarrow_{\mathcal{C}} a$ admits a cofinal functor from a finite ∞-category. This condition implies that for a stable presentable ∞-category $\mathcal{V}$, the inclusion of the full subcategory of $\mathrm{Fun}(\mathcal{C}, \mathcal{V})$ spanned by functors that are left Kan extensions from $\mathcal{C}_\alpha$ admits a left adjoint $\mathbf{P}_\alpha$. We say that the functors in this subcategory are α-polynomial.

This simple structure is enough to establish the existence of "Taylor systems" (not necessarily a tower, since the elements of Π may not be totally ordered under inclusion) of universal approximations by polynomial functors. Moreover, this rudimentary structure also furnishes a classification of the homogeneous functors by Taylor coefficients: for $\mathcal{C}_\alpha, \mathcal{C}_\beta \in \Pi$, the ∞-category of β-polynomial functors E such that $\mathbf{P}_\alpha E = 0$ is equivalent to the functor ∞-category $\mathrm{Fun}(\mathcal{C}_\beta \setminus \mathcal{C}_\alpha, \mathcal{V})$. And corepresentation calculi can be proliferated along Cartesian fibrations: given a

Cartesian fibration $\varpi : \mathcal{D} \to \mathcal{C}$ and a corepresentation calculus on $\mathcal{C}$, the preimage subcategories $\varpi^{-1}(\mathcal{C}_\alpha)$ determine a corepresentation calculus on $\mathcal{D}$.

Aside from its progenitor, FI-calculus, several other functor calculi in the literature fit into the framework of corepresentation functor calculus when their codomain ∞-categories are stable and presentable. By a theorem of Lurie, [Lur17, Theorem 6.1.5.6], the Goodwillie calculus for functors $E : \mathcal{PC} \to \mathcal{D}$ is a corepresentation calculus, where $\mathcal{PC}$ denotes the presheaf ∞-category of some small ∞-category $\mathcal{C}$; and by a theorem of Barnes, Kędziorek, and Taggart, [BKT25, Theorem B], orthogonal calculus is a corepresentation calculus.

With more assumptions on $\mathcal{C}$, more qualities of the associated functor calculus can be deduced. If the elements of Π are downward closed, meaning that for all $a \in \mathcal{C}_\alpha$ and $f : b \to a$, one has $b \in \mathcal{C}_\alpha$, then the Taylor coefficients of representable functors are given by a simple formula, allowing for the calculation of the natural transformations between the Taylor coefficient functors. In ongoing work, I hope to show that the adjunction between Taylor systems and Taylor coefficients is comonadic. The accumulation of niceness conditions on domain ∞-categories for corepresentation calculus culminates in the definition of "copacetic categories".

Definition. *A 1-category is* copacetic *if for each $a \in \mathcal{C}$, only finitely many isomorphism classes admit morphisms to a, all hom-sets in $\mathcal{C}$ are finite, all endomorphisms in $\mathcal{C}$ are automorphisms, and for each $b \in \mathcal{C}$, the group* $\operatorname{Aut} a$ *acts freely on $\mathcal{C}(a, b)$.*

Theorem. *If a 1-category $\mathcal{C}$ is copacetic, then it admits a corepresentation calculus in which Π consists of all downward-closed subcategories with finitely many isomorphism classes and the Taylor coefficients of a functor $\mathcal{C} \to \mathcal{V}$ for $\mathcal{V}$ a stable presentable ∞-category again carry the structure of a functor $\mathcal{C} \to \mathcal{V}$.*

When Tate constructions in $\mathcal{V}$ vanish, taking Taylor coefficients yields an equivalence of ∞-categories between analytic functors from $\mathcal{C}$ to $\mathcal{V}$ and torsion functors from $\mathcal{C}$ to $\mathcal{V}$.

To the best of my knowledge, all domain categories considered in the homological stability and representation stability literature, including Gadish's categories of FI-type, introduced in [Gad17]; $\mathsf{FS}^{\mathrm{op}}$, studied by Tosteson in [Tos22]; the homogeneous categories considered by Randal-Williams and Wahl in [RW17]; and (assuming a mild finiteness condition) the weak complemented categories of Putman and Sam in [PS17] are copacetic. In work in progress, I generalize my results relating FI-calculus and representation stability to link polynomial functors on copacetic categories to the corresponding notions of homological and representation stability.

Functor calculus in an ∞-equipment. My second main project is the development of a framework for functor calculus in an ∞-equipment. The heart of the approach lies in the observation that for a given functor calculus, a domain ∞-category x, a codomain ∞-category y, and a notion of α-polynomial functor $x \to y$ with polynomial approximation given by $P_\alpha : \operatorname{Fun}(x, y) \to \operatorname{Fun}(x, y)$, there exists some profunctor $W : x \nrightarrow x$ (in fact, an idempotent comonad in the $(\infty, 2)$-category of small ∞-categories and profunctors) such that α-polynomial approximation of an arbitrary functor $f : x \to y$ is given by the W-weighted limit of f: $P_\alpha \cong lim^W$.

This perspective on polynomiality provides a streamlined approach to defining functor calculus in an ∞-equipment. Given an idempotent proarrow comonad W on an object x in the horizontal fragment $\operatorname{Hor}(\mathcal{P})$ of an ∞-equipment $\mathcal{P}$, an arbitrary object $y \in \mathcal{P}$, and an arrow $f : x \to y$, we say that f is W-polynomial if

$\lim^W f$ exists and the morphism $f \to \lim^W f$ determined by the counit of W is an isomorphism. When y admits all W-weighted limits, the idempotent monad $\lim^W$: $\mathrm{Vert}(\mathcal{P})(x,y) \to \mathrm{Vert}(\mathcal{P})(x,y)$ provides reflection into the subcategory $\mathrm{Poly}_W(y)$ of W-polynomial arrows. An idempotent comonad U determines a "lesser" degree of polynomiality than W when U is a (W,W)-bicomodule.

We can also easily recover a key phenomenon from corepresentation calculus: when W is the comonad of an adjunction

$$z \underset{R}{\overset{L}{\rightleftarrows}} x \quad (L \dashv R)$$

such that the unit of the adjunction is an isomorphism – in other words, when W admits a coKleisli object z in $\mathrm{Hor}(\mathcal{P})$ – then, assuming y admits the relevant weighted limits, we obtain a classification $\mathrm{Vert}(\mathcal{P})(z,y) \cong \mathrm{Poly}_W(y)$, and if R is the conjoint $g^{\circledast}$ of an arrow $g : z \to x$, then this equivalence is given by left Kan extension

$$\mathrm{Lan}_g : \mathrm{Vert}(\mathcal{P})(z,y) \cong \mathrm{Poly}_W(y)$$

as in a corepresentation calculus. A similar but slightly more convoluted construction allows us to encode the classification of homogeneous arrows in this language.

In a corepresentation calculus with downward closed distinguished subcategories, extending a system of Taylor coefficients to a formal Taylor system involves certain lifting problems along (generalized) norm maps relating the (weighted) colimits and limits of diagrams. In a forthcoming note, I define these norm maps in the context of an ∞-equipment and show that they are instantiated by Poincaré duality, the Adams isomorphism of equivariant homotopy theory [CLL24], the u-right stability of Rahn and Shulman [RS21], the dualizing spectra of Klein [Kle07], and other morphisms which canonically relate colimits and limits.

If the ∞-equipment $\mathcal{P}$ is closed monoidal, it is possible to define internal mapping objects $\underline{\mathrm{Poly}}_W(y)$ of W-polynomial arrows for a given idempotent proarrow comonad W. I plan to generalize the Bauer–Burke–Ching–Goodwillie tangent structure by giving axioms for W sufficient to endow $\underline{\mathrm{Poly}}_W(-)$ with the structure of a tangent bundle functor as Bauer, Burke, and Ching do for 1-excisive functors out of $\mathcal{S}_{\mathrm{fin}}$, but that work is currently incomplete.

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